M.Sc (Mathematics) — (Sem III)

EC Math 304.

Model Question Paper.

Very Short answer type Questions:

1) (a) Determine the acceleration of a fluid particle from the following flow field

$$q = i(Axy^2t) + j(Bx^2yt) + k(Cxyz).$$

(b) obtain the streamlines of flow $u = x$, $v = -y$.

(c) Define streamlines and path lines of a particle.

(d) Write the equation of Continuity of an incompressible fluid.

(e) Write the Condition that the surface $F(r,t) = 0$ or $F(x,y,z,t) = 0$ may be a boundary surface.

(f) Give examples of irrotational and rotational flow.

(g) Write equation of Continuity by Euler's method.

(h) The Velocity potential function for a two dimensional flow is $\phi = x(2y-1)$ at a point $P(4,5)$ find (i) the Velocity and (ii) the value of stream function.

(i) Show that $u = 2cxy$ & (3)

$$v = c(a^2 + x^2 - y^2)$$

are the velocity components of a possible fluid motion.

(4)

(j) Define Sources and Sink.

(k) If w is an analytic function of z where w is known as a complex potential then write the Cauchy-Riemann equation in polar form.

(5)

(l) Write the statement of the Milne-Thomson Circle theorem.

(6)

Short Answer type Questions.

(1) Describe the Lagrange's and Eulerian methods of describing the fluid flows and distinguish between them.

(2) Show that the surface

$$\frac{x^2}{a^2k^2t^4} + kt^2\left(\frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 1$$

is a possible form of boundary surface of a liquid at time t.

(3) Find the stream lines and Paths of the particles when

$$u = \frac{x}{(1+t)}, \quad v = \frac{y}{(1+t)} \quad = \frac{z}{(1+t)}$$

(4) Show that the motion of an incompressible fluid moving irrotationally is given by $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$

where ϕ is the velocity potential.

(5) Obtain Euler's equation of motion in Cartesian form.

(6) Let there be a source of strength m at $(a,0)$ and sink $-m$ at $(0,a)$. Find ϕ, ψ & w.

(7) A two - dimensional flow flied is given by $\psi = xy$. (a) Show that the flow is irrotational

(b) Find the velocity potential

(c) Verify that ψ & ϕ satisfy the Laplace equation

(d) Find the streamlines and potential lines.

(8) There is a source of strength m at $(0,0)$ and equal sinks at $(1,0)$ & $(-1,0)$. Discuss two dimensional motion. Also draw the stream lines.

Long Answer Type Questions.

(1) Define equation of Continuity and derive the equation of Continuity in spherical polar Co-ordinate.

(2) Derive the equation of Continuity by Lagrangian method and establish the equivalence relation between Eulerian and Lagrangian form of equation of Continuity.

(3) Obtain Bernoulli's equation for the steady motion.

(4) Find the stream function of the two dimensional motion due to two equal sources and an equal sink situated midway between them.

(5) Show that if the velocity potential of an irrotational fluid motion is equal to

$$A\left(x^2+y^2+z^2\right)^{-3/2} z \cdot \tan^{-1}\left(\frac{y}{x}\right)$$

(6) What arrangement of sources and sinks will give rise to the function

$$w = \log\left(z - \frac{a^2}{z}\right).$$

Draw a rough sketch of the streamlines. Prove that the two of the streamlines subdivide into the circle

$r = a$ & axis of y.